CALCULATING THE PARAMETERS OF A CYLINDRICAL ARC WITHOUT APPROXIMATION OF THE ELECTRICAL CONDUCTIVITY AND THE RADIATION ENERGY

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The parameters of an arc are calculated approximating neither the electrical conductivity nor the radiation energy. The results of these calculations are compared with experimental data.

The energy equation for an arc column was solved in [1] without approximating the electrical conductivity but considering thermal conduction to account for the dissipation of electric power. It appears worthwhile to solve a more complete equation where both thermal conduction and radiation are considered to account for dissipation of electric power without resorting to approximation of the electrical conductivity or of the radiation energy. Such a calculation is of practical value, since it will allow one to plot a volt–ampere characteristic without the cumbersome computations involved in approximating the electrical conductivity and the radiation energy. The thermal conductivity is usually approximated by introducing the heat-flux function S.

A basic difficulty in the problem is the determination of this heat-flux function $S_1(r)$ from the energy equation, r being here the radial coordinate of the arc column. Assuming that the difference between the heat source $q(S_1(r))$ and the radiation energy $U(S_1(r))$ is linear with respect to $S_1(r)$, it will be shown that this function can be expressed in terms of a Bessel function. In [2] Maecker also expressed $S_1(r)$ in terms of a Bessel function, but he assumed the energy source to be linear with respect to $S_1(r)$. The linearity of the source was based on a linear approximation of the electrical conductivity. Radiation was not taken into account there.

In our case the difference $q(S_1(r))-U(S_1(r))$ is assumed to be a linear function of $S_1(r)$ while the radiation energy $U(S_1(r))$ – as long as no approximation has been made – is a nonlinear function of $S_1(r)$, and, therefore, the heat source $q(S_1(r))$ becomes nonlinear with respect to $S_1(r)$. Furthermore, in the given problem, the entire cylindrical channel of radius R is filled with a conducting gas and the boundary conditions, i.e. the heat-flux power $S_1(R)$ as well as the heat-flux gradient $S_{1r}(R) = dS_1/dr|_{r=R}$ at the channel wall are stipulated. The heat-flux function $S_1(r)$ is now found from the energy equation for an arc column:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS_1}{dr}\right)+q(S_1(r))-U(S_1(r))=0 \tag{1}$$

with the boundary conditions

$$S_1(R)=0,\ S_{1r}(R)=S_*\frac{1}{R\ln\dfrac{R}{R+\delta}}, \tag{2}$$

where δ is the thickness of the channel wall, across which there is a temperature gradient, and S_* is the heat-flux function at the channel wall.

The value of S_* is calculated from the equation

$$S_* = \lambda_1(T_* - T_0), \tag{3}$$

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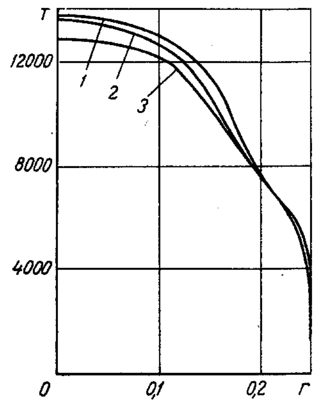

Fig. 1

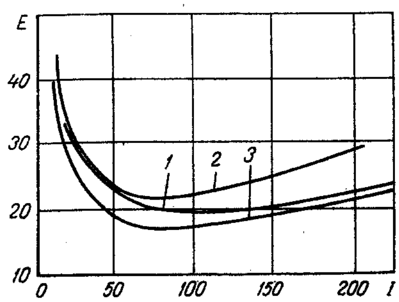

Fig. 2

Fig. 1. Temperature distribution T(r) (I = 100 A, E = 22 V/cm): 1) theoretical curve according to the method given here; 2) the same according to [3]; 3) experimental curve according to [5]. T (°K), r (cm).

Fig. 2. Volt–ampere characteristics of an arc column (E, V/cm; I, A) for d = 0.5 cm: 1) theoretical curve according to the method given here; 2) experimental curve according to [4]; 3) theoretical curve according to [3].

where λ_1 is the thermal conductivity of copper; T_* is the temperature of the channel wall on the plasma side (lower than the melting point of copper); and T_0 is the temperature at $r \geq R + \delta$, equal to the temperature of the cooling water.

The temperatures T_* and T_0 are chosen as the boundary conditions for the equation of heat conduction within the region δ. This equation is analogous to the equation of heat conduction for the nonconducting region of the arc column [1, 2].

The boundary conditions (2) have been obtained from the solution to the equation of heat conduction within the region δ. Various gradients (2) of the heat-flux function at the boundary of the arc column, which are needed for calculating the volt–ampere characteristic, have been obtained by varying the size of region δ.

As the intensity of cooling the channel wall to temperature T_0 increases, the gradient of the arc heat flux also increases and this corresponds to an increasing electric current. An important feature of the problem here is also that the arc column is not subdivided into a conducting and a nonconducting region, and that the electrical conductivity $\sigma(T_*)$ of the plasma adjacent to the channel wall is determined from T on the basis of the relation between electrical conductivity σ and temperature T.

We now let

$$q(S_1(r)) - U(S_1(r)) = \frac{p^2}{R^2} S_1(r), \tag{4}$$

where p = 2.403 is the smallest root of the Bessel function $J_0(x)$.

Then Eq. (1) becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS_1}{dr}\right) + \frac{p^2}{R^2} S_1(r) = 0, \tag{5}$$

the solution of which under conditions (2) is

$$S_1(r) = S_* \frac{J_0\left(\frac{p}{R} r\right)}{pJ_1(p) \ln \frac{R+\delta}{R}}, \tag{6}$$

with

$$J_1(p) = 0.52.$$

In order to proceed with the calculations, it is necessary to integrate the thermal conductivity of the plasma λ with respect to T between T_* and T. As a result, we obtain an expression for $S_1(T)$ which allows us to express the radiation energy U(T) in terms of S_1.

Knowing the distribution of $S_1(r)$, we express the radiation energy $U(S_1)$ as a function of r. From this then, according to (4), we find the heat-source distribution q(r). The total arc energy will be

$$IE = 2\pi \left(S_* \frac{1}{\ln \frac{R+\delta}{R}} + \int_0^R U(r)\, rdr \right). \quad (7)$$

The temperature T(r) is found from the $S_1(T)$ curve based on $S_1(r)$, while the electrical conductivity $\sigma(r)$ is found from the $\sigma(T)$ curve based on T(r).

Furthermore, we have the equation

$$\frac{I}{E} = 2\pi \int_0^R \sigma(r)\, rdr, \quad (8)$$

where $\sigma(r)$ is treated without approximation. With the aid of Eqs. (7) and (8), one can determine the current I and the electric field intensity E. Knowing $\sigma(r)$ and E, one can then determine the current-density distribution, express the heat source in terms of σE^2, and represent Eq. (1) in the form:

$$\frac{1}{r}\frac{d}{dr}\left(r \frac{dS_1}{dr} \right) - U(S_1) + \sigma E^2 = 0. \quad (9)$$

This emphasizes that the Joule heat given off in the arc is generated by an electric current.

Calculations by this method were made for an arc in nitrogen, then a comparison was made with calculated data in [3] and experimental data in [4, 5]. The results obtained here, i.e. the temperature T(r) and the volt–ampere E(I) characteristics are shown in Figs. 1 and 2.

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